



## Early Journal Content on JSTOR, Free to Anyone in the World

This article is one of nearly 500,000 scholarly works digitized and made freely available to everyone in the world by JSTOR.

Known as the Early Journal Content, this set of works include research articles, news, letters, and other writings published in more than 200 of the oldest leading academic journals. The works date from the mid-seventeenth to the early twentieth centuries.

We encourage people to read and share the Early Journal Content openly and to tell others that this resource exists. People may post this content online or redistribute in any way for non-commercial purposes.

Read more about Early Journal Content at <http://about.jstor.org/participate-jstor/individuals/early-journal-content>.

JSTOR is a digital library of academic journals, books, and primary source objects. JSTOR helps people discover, use, and build upon a wide range of content through a powerful research and teaching platform, and preserves this content for future generations. JSTOR is part of ITHAKA, a not-for-profit organization that also includes Ithaka S+R and Portico. For more information about JSTOR, please contact [support@jstor.org](mailto:support@jstor.org).

$$\Delta = \begin{vmatrix} -\frac{yz}{x^2}, & \frac{z}{x}, & \frac{y}{x} \\ \frac{z}{y}, & -\frac{xz}{y^2}, & \frac{x}{y} \\ \frac{y}{z}, & \frac{x}{z}, & -\frac{xy}{z^2} \end{vmatrix} = xyz \begin{vmatrix} -\frac{1}{x}, & \frac{1}{y}, & \frac{1}{z} \\ \frac{1}{x}, & -\frac{1}{y}, & \frac{1}{z} \\ \frac{1}{x}, & \frac{1}{y}, & -\frac{1}{z} \end{vmatrix} = xyz \begin{vmatrix} 0, & 0, & \frac{2}{z} \\ \frac{2}{x}, & 0, & 0 \\ \frac{1}{x}, & \frac{1}{y}, & -\frac{1}{z} \end{vmatrix} = 4.$$

The value announced,  $4xyz$ , is therefore erroneous.

Solved similarly by J. Scheffer and W. J. Greenstreet.

---

### MECHANICS.

---

356. Proposed by the late G. B. M. ZERR, Ph. D.

A cantilever beam length  $a$  is loaded with  $c$  pounds per running foot at its fixed end and increases uniformly to  $b$  pounds per running foot at its free end. Find the deflection at the free end due to this load.

Solution by FRANCIS RUST, E. E., Pittsburg, Pa.

The problem is solved by determining the elastic curve of the beam's axis from its second differential equation

$$y'' = \frac{M}{IE},$$

as derived from Hooke's law, to be found in all text books.  $M$  is the bending moment in the point, abscissa= $x$ ;  $I$  is the moment of inertia of the beam's cross-section, and  $E$  is the modulus of elasticity of its material.

$M$  is to be determined by the arrangement of the load with mathematical certainty. Taking the point, abscissa= $x$ , for origin,

$$M = \int_0^{a-x} q \xi d\xi,$$

$q$ , the load per lineal foot, to be expressed as a function of the variable of integration  $\xi$ .  $q = c + px$ , and  $c + pa = b$ . Consequently,

$$p = \frac{b-c}{a},$$

and  $q$  as a function of  $\xi$  is

$$q=c+\frac{b-c}{a} \cdot (x+\xi) = \frac{ac+(b-c)x}{a} + \frac{b-c}{a} \xi, \text{ and}$$

$$\begin{aligned} M &= \frac{ac+(b-c)x}{a} \int_0^{a-x} \xi d\xi + \frac{b-c}{a} \int_0^{a-x} \xi^2 d\xi = \frac{ac+(b-c)x}{a} \cdot \frac{(a-x)^2}{2} \\ &\quad + \frac{b-c}{a} \cdot \frac{(a-x)^3}{3} = \frac{a^2}{6} \left( 1 - \frac{x}{a} \right)^2 \left( (2b+c) + (b-c) \frac{x}{a} \right) \end{aligned}$$

which, properly simplified, gives us

$$M = \frac{a^2}{6} \left[ (2b+c) - 3(b+c) \frac{x}{a} + 3c \left( \frac{x}{a} \right)^2 + (b-c) \left( \frac{x}{a} \right)^3 \right].$$

Assuming now a beam of uniform cross-section and homogeneous material, and introducing the new variable,  $x/a=u$ , we have  $dx=adu$ .

The integration of  $y''=M/IE$  may be performed without any difficulty. It gives the equation of the elastic curve,

$$y = \frac{a^4}{6EI} \left[ \frac{2b+c}{2} \left( \frac{x}{a} \right)^2 - \frac{b+c}{2} \left( \frac{x}{a} \right)^3 + \frac{c}{4} \left( \frac{x}{a} \right)^4 + \frac{b+c}{20} \left( \frac{x}{a} \right)^5 \right]$$

from which is derived for  $x=a$ ,

$$y_a=f=\frac{a^4(11b+4c)}{120EI}.$$

*Remark.* This result is as simple as it is wrong.

In my memoir *Der Fehler in Hooke's Gesetz*, published in Oesten, Wochensch. f. d. aff. Bandienst, year 1900, p. 252, ff., I have deduced mathematically and proved by experiment, that Hooke's law *cannot be applied on the beginning of deformation*.

The locus of any elastic curve, deduced from Hooke's law, *p. e.* our equation above, is a continuous curve, whilst the experiment shows, that a cantilever (spring under its own weight, for instance) will remain *unbent for a certain length at its free end*.

The investigation into these conditions reveals the fact, that any material may be submitted to a certain stress, which must be overcome before deformation takes place. Such minimal stress may be called *the resistance of inertia* (Tragheits widerstand). Denoting it by  $\alpha$ , the true expression of the law of elastic deformation is

$$\triangle l:l=\sqrt{(\sigma^2+\alpha^2)}:E.$$